Preprint typeset in JHEP style - HYPER VERSION

hep-ph/0606107, LAPTH-1149/06

Can isocurvature bounds rule out the axion?

María Beltrán[1], **Juan García-Bellido**[1] **and Julien Lesgourgues**[2]

[1] *Departamento de Física Teórica C-XI, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain*

[2] *Laboratoire de Physique Théorique LAPTH, F-74941 Annecy-le-Vieux Cedex, France*

E-mail: maria.beltran@uam.es, juan.garciabellido@uam.es, lesgourg@lapp.in2p3.fr

Abstract: The axion is one of the best motivated candidates for particle dark matter. Its predicted contribution to the matter content of the universe makes it worth looking for in axion search experiments like ADMX and CAST. However, under very general conditions, axions produced by the misalignment mechanism during inflation will also induce too large isocurvature perturbations, which can be strongly constrained by recent observations of the CMB and LSS. In particular, for inflation taking place at intermediate or high energy scales, we derive some restrictive limits which can only be evaded by assuming an efficient reheating mechanism, with $T_{\rm rh} > 10^{11}$ GeV. Chaotic inflation with a quadratic potential is still marginally compatible with the axion scenario, provided that the Peccei-Quinn scale f_a is close to 10^{10} or 10^{11} GeV and the reheating temperature is slighly above, $T_{\rm rh} > f_a$. Isocurvature bounds eliminate the possibility of a larger f_a and a small misalignment angle. Isocurvature constraints on the axion scenario must be taken into account whenever the scale of inflation is above 10^{12} GeV; below this scale, axionic isocurvature modes are too small to be probed by current observations.

Keywords: axion, dark matter, isocurvature perturbations.

1. Introduction

The fact that the strong sector of the Standard Model conserves the discrete symmetries P and CP while the electroweak sector doesn't, also known as the *strong CP problem*, is considered a serious puzzle for modern particle physics [1]. The most elegant and compelling solution to this problem was proposed in 1977 by Peccei and Quinn [2] with the introduction of a new $U(1)_{\rm PQ}$ global symmetry at high energies. The symmetry is spontaneously broken in the early universe and the resulting Nambu-Goldstone boson is known as the *axion* [3]. Non perturbative effects at the QCD scale give a potential to the axion, whose minimum eliminates the CP-violating terms, thus agreeing with the observed electric dipole moment of the neutron [4]. Even though the existence of the axion was postulated soon after the Peccei-Quinn proposal this particle has never been detected by any direct or indirect searches, see e.g. [5].

The axion mass and couplings to the rest of the matter are inversely proportional to the scale, f_a, of $U(1)_{\rm PQ}$ symmetry breaking [6, 5]. Indeed, the mass is found to be

$$m_a = \frac{\sqrt{z}}{1+z}\,\frac{f_\pi m_\pi}{f_a} = 6.2\,\mu{\rm eV}\left(\frac{10^{12}\ {\rm GeV}}{f_a}\right)\,, \tag{1.1}$$

where $z = m_u/m_d \simeq 0.56$ is the mass ratio of up to down quarks, and m_π, f_π are respectively the pion mass and decay constant. The generic tree-level coupling to photons γ and fermions f reads

$$\mathcal{L}_{\rm int} = g_\gamma\,\frac{\alpha}{\pi}\,\frac{a(x)}{f_a}\,\vec{E}\cdot\vec{B} + i\,g_f\,\frac{m_f}{f_a}\,a(x)\,\bar{f}\,\gamma_5\,f\,, \tag{1.2}$$

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields, α is the electromagnetic coupling constant, and for each species g_i is a model-dependent coefficient of order one. The different ways in which the symmetry is accomplished, and then broken, give rise to different axion models that forecast similar axion properties. Nevertheless, there exists one remarkable distinction between them, which is the predicted coupling to electrons; for the "hadronic" models, such as the KSVZ model [7], one has $g_e = 0$, while the tree-level coupling does not vanish for the non-hadronic DFSZ models [8]. The other couplings are of the same order. For example, $g_\gamma = -0.36$ in the DFSZ model, while $g_\gamma = 0.97$ in the KSVZ model.

In the original axion model a physical meaning was given to the symmetry breaking scale by fixing it to the electroweak scale, i.e. $f_a \sim 100$ GeV, which was soon ruled out by direct searches (see, for example [1, 5]). In the currently accepted *invisible axion* model, the scale f_a is in principle arbitrary, well above the electroweak scale so that the axion coupling to matter is weak enough to pass undetected, for the moment. There are at present several experiments searching for the axion in the laboratory, like ADMX [9] and CAST [10], which have recently reported bounds on the axion coupling to matter [11].

Moreover, such a field acts like a weakly interacting massive particle and could constitute today's dark matter. In fact, the axion is, together with primordial black holes, the best non-supersymmetric candidate for particle dark matter. It could have the correct relic density $\rho_{\rm cdm}$ today for matching constraints from Cosmic Microwave Background (CMB) and Large Scale Structure (LSS) observations, and its velocity is so low that it constitutes a non-relativistic component, in spite of its low mass. In fact, its Compton and de Broglie wavelengths are

$$\lambda_{\rm C} = \frac{\hbar}{m_a} = 20\,{\rm cm}\left(\frac{1\,\mu{\rm eV}}{m_a}\right)\,, \qquad\qquad \lambda_{\rm dB} = \frac{\hbar c}{p_a} = 1\,{\rm pc}\left(\frac{1\,\mu{\rm eV}}{m_a}\right)^{0.18}\,, \tag{1.3}$$

which are typically smaller than the scales on which dark matter is expected to cluster, and larger than the classical inter-particle distance $\lambda_{\rm cl} = (m_a/\rho_{\rm cdm})^{1/3}$. The axion is thus a Bose condensate with large occupation numbers, but on relevant cosmological scales it behaves like particle dark matter.

Also, since in the small coupling (large f_a) limit the axion remains effectively decoupled from the rest of the particle species, its fluctuations during inflation could induce isocurvature perturbations in the CMB anisotropies spectrum which would in principle be observable today [12, 13, 14, 15, 16, 17, 18]. We will take into account these two facts to put strong constraints on its relic density and its mass today.

The paper is organized as follows: in Section 2 we review the different production mechanisms, focusing on the misalignment angle and the consequences of the De Sitter stage during inflation; in Section 3 we study the induced isocurvature component; in Section 4 we present the additional constraints bounding the axionic window; finally, our results and conclusions are described in sections 5 and 6.

2. Production mechanisms

As pointed out in the introduction, the Peccei-Quinn global $U(1)_{PQ}$ symmetry is spontaneously broken by some scalar symmetry breaking field $\psi = \rho/\sqrt{2}\,e^{i\Theta}$. The symmetry breaking potential is

$$V(\psi) = \frac{\lambda}{4}\left(\rho^2 - \frac{f_a^2}{N^2}\right)^2 = V_{\rm PQ} - \frac{1}{2}m_\psi^2\rho^2 + \frac{\lambda}{4}\rho^4\,, \tag{2.1}$$

where N is the number of degenerate QCD vacua associated with the color anomaly of the PQ symmetry. Spontaneous symmetry breaking (SSB) occurs when the energy density of the universe falls below $V_{\rm PQ}$ and the field acquires a vacuum expectation value (VEV), $\rho = f_a/N$. Note, however, that SSB is effective only when the typical fluctuations on $\delta\rho$ are smaller than f_a/N. If either T or $H_{\rm inf}$ are of order f_a/N at reheating or during inflation, thermal [19] or quantum [20] fluctuations (respectively), will modify the effective potential and restore the PQ symmetry.

Let us focus now on the case where no symmetry restoration occurs, and the radial part of the field oscillates perturbatively around the value f_a/N after the symmetry is broken (we will come back to the case with symmetry restoration in later sections). The phase of the field, Θ, moves along the flat direction of the potential and remains massless. This is the Goldstone boson of the PQ symmetry breaking associated to a residual global $U(1)$ symmetry of the theory [1, 3]. The axion is related to the phase of the PQ field by

$$a(\vec{x}) = \frac{f_a}{N}\Theta(\vec{x})\,. \tag{2.2}$$

As the universe expands, and its energy decreases to about $\Lambda_{\rm QCD}$, the non-perturvative instanton effects *tilt* the previously flat potential and the phase symmetry is explicitly broken [4]. The new induced potential is

$$V(a) \simeq m_a^2 \frac{f_a^2}{N^2}(1-\cos\Theta) \tag{2.3}$$

which is obviously no longer flat. The axion field acquires a mass about the minimum of the potential that depends on the temperature in the vicinity of $T \sim \Lambda_{\rm QCD}$ as [21, 22]

$$m_a(T) \simeq m_a C \left(\frac{\Lambda_{\rm QCD}}{200{\rm MeV}}\right)^{\frac{1}{2}} \left(\frac{\Lambda_{\rm QCD}}{T}\right)^4 , \tag{2.4}$$

where m_a is the zero temperature axion mass (1.1) and C is a model-dependent factor calculated in Refs. [21, 22] to be of the order of $C \simeq 0.018$. The field equation of motion is

$$\ddot{a} + 3H\dot{a} + V'(a) + \frac{1}{R^2}\nabla^2 a = 0\,, \tag{2.5}$$

where $V'(a) = \partial V/\partial a$, with ∇^2 the comoving laplacian, and R is the scale factor of the universe. If the axion field is initially sufficiently close to the minimum of the potential when it acquires its mass, then $V'(a) \simeq m_a^2 a$. Otherwise, anharmonic effects should be taken into account by inserting in (2.5) the actual instanton contribution to the mass of the axion. In either case, the mass and the potential term are time-dependent.

Axions are produced in the early Universe by various mechanisms. Any combination of them could be the one responsible for the present axion abundance. We will briefly describe here the different scenarios and production mechanisms. For detailed reviews see Refs. [1, 5].

2.1 Thermal production

If the coupling of axions to other species is strong enough (i.e. f_a low enough), then axions may be produced from the plasma in the early universe and it is possible that an axionic thermal population existed at high energy. If this is the case, a relic density of thermally generated axions would be present nowadays and could significantly contribute to the current cold dark matter component of the universe.

The two main processes that dominate the thermal production of axions are photo- and gluon-production and pion-axion conversion (the axion and the pion share the same quantum numbers and thus they can oscillate into each other). Since nucleons and mesons only exist after the quark-hadron transition, the second mechanism is only posible after $T \sim \Lambda_{\rm QCD} \simeq 200$ MeV. During the thermal equilibrium stage, the axion density normalized to the entropy of the universe ($Y \equiv n_a/s$) acquires an equilibrium value $Y_{\rm eq}$. Since it is assumed that the axion is relativistic during the epoch of interest, we have

$$Y_{\rm eq} = \frac{\zeta(3)T^3/\pi^2}{(2\pi^2/45)\ g_{*,\rm eq}T^3} \simeq \frac{0.27}{g_{*,\rm eq}}\,, \tag{2.6}$$

where $g_{*,\mathrm{eq}}$ is the number of relativistic degrees of freedom at the temperature where the axions reach their equilibrium distribution. The decoupling of axions from thermal equilibrium is described by the Boltzmann equation [23]:

$$Y' = \left(\frac{\Gamma_{\mathrm{abs}}}{xH}\right)(Y_{\mathrm{eq}} - Y)\,, \tag{2.7}$$

where $x^{-1} \equiv T/m$ represents the temperature normalized to a convenient energy scale $m \sim \Lambda_{\mathrm{QCD}}$, H is the Hubble expansion rate, and Γ_{abs} is the thermal-averaged interaction rate for the process $a + i \leftrightarrow 1 + 2$, see Ref. [24]. Since Γ_{abs}/H decreases and acquires values < 1 for $T < \Lambda_{\mathrm{QCD}}$, the axions cease to be in thermal equilibrium with the rest of the species and they freeze out when the rate of the interactions cannot keep up with the expansion of the universe. More precisely, it is shown in Ref. [23] that Γ_{abs}/H presents a peak at the QCD scale and then decays exponentially. So, it is possible to find the relic abundance of thermal axions by integrating forward in time from $T \simeq 200$ MeV till today the solution to (2.7),

$$Y(x) = Y_{\mathrm{eq}}\left(1 - \exp\left[-\int_0^x \frac{\Gamma_{\mathrm{abs}}}{x'H}dx'\right]\right)\,. \tag{2.8}$$

Finally, the relic abundance of axions can be written as

$$Y_\infty = \frac{0.278}{g_{*,F}}\left(1 - \exp\left[-\left(\frac{m_a}{10^{-4}\,\mathrm{eV}}\right)^2 x_{\mathrm{qh}}^{-5/2}\, e^{-x_{\mathrm{qh}}}\right)\right]\right)\,, \tag{2.9}$$

which depends on the number of relativistic degrees of freedom at the freeze-out temperature $g_{*,F}$ (and not at the equilibrium temperature) and on $x_{\mathrm{qh}} = m/\Lambda_{\mathrm{QCD}}$. We can thus extract the number density of thermal axions and their contribution to the matter density of the universe,

$$\Omega_{\mathrm{a}}^{\mathrm{ther}} h^2 = \frac{m_a}{130\,\mathrm{eV}}\left(\frac{10}{g_{*,F}}\right)\,, \tag{2.10}$$

with $\Omega_{\mathrm{a}} \equiv \rho_a/\rho_c = 8\pi G\rho_a/3H_0^2$. This result applies only when $m_a > 10^{-3}$ eV: otherwise, the peak value of Γ_{abs}/H at the QCD scale is smaller than one, and the axions never reach thermal equilibrium. On the other hand, the current WMAP bound on the cold dark matter density [25]

$$\Omega_{\mathrm{cdm}} h^2 = 0.112^{+0.003}_{-0.006}\,,$$

together with Eq. (2.10), imposes a bound on the axion mass $m_a < 14.5$ eV. As we will see in Section 4.2, this bound is overseeded by astrophysical data which forbid a mass range of $0.01\,\mathrm{eV} < \mathrm{m_a} < 200$ keV for the DFSZ axion [1, 5].

Hadronic axions are not so tigthly constrained by astrophysical data because they do not take part (at tree level) in the processes that cause the anomalous energy loses in stars such as $\gamma + e^- \to e^- + a$ or the Primakoff effect [1]. Thus, a narrow mass window remained open until recently for thermal hadronic axions of $m_a \simeq$ several eV. However, a more detailed look at the freeze out temperature and a combination of cosmological data from CMB and LSS has allowed Ref. [26] to shut the window on the axion mass, $m_a < 1.05$ eV, in a model independent way. We will therefore ignore from now on the thermal axion contribution to cold dark matter.

2.2 Axion production via cosmic strings

We already mentioned that the PQ symmetry could be restored at high energy, e.g. by large quantum fluctuations during inflation, or large thermal fluctuations after reheating. After each symmetry restoration phase, the spontaneous symmetry breaking (SSB) will produce a population of axionic cosmic strings. Let us review the three cases in which axionic strings are produced after, during or before inflation:

- *after inflation:* if the scale f_a/N is below the reheating temperature of the universe, the PQ symmetry is restored at reheating. Then, axionic cosmic strings will be produced when the temperature of the Universe falls below f_a/N [19]. These strings typically decay into axion particles before dominating the energy density of the universe. The axions produced this way are relativistic until the QCD transition, where they acquire a mass and become non-relativistic. Eventually these axions may come to dominate the energy density after equality, in the form of cold dark matter. Estimates of their present energy density vary depending on the fraction of axions radiated by long strings versus string loops. Three groups have studied this issue and found agreement within an order of magnitude [27, 19, 28],

$$\Omega_{\rm a}^{\rm str} h^2 \simeq 4\Delta_{\rm QCD} \left(\frac{1\mu{\rm eV}}{m_a}\right)^{1.18} \tag{2.11}$$

  where $\Delta_{\rm QCD} = 3^{\pm 1}$ is a "fudge factor" which takes into account all the uncertainties in the QCD phase transition. Similar bounds were found in [29] following a very different approach.

- *at the end of inflation:* the PQ symmetry is restored during inflation whenever the typical amplitude of quantum fluctuations $H_{\rm inf}/2\pi$ exceeds the symmetry breaking scale f_a/N [20]. If the inequality

$$\frac{H_{\rm inf}}{2\pi} > \frac{f_a}{N} \tag{2.12}$$

  holds throughout inflation, comic strings will be produced at the very end of this stage. The mechanism by which cosmic strings are produced at the end of inflation is very different from that of a thermal phase transition and could affect the number of infinite strings, and thus the approach to the scaling limit, with the subsequent estimate of the relic density of axions. If, after all, the scaling limit is approached, then the present energy density of axion will be approximately given by Eq. (2.11).

- *during inflation:* the PQ SSB could occur during inflation, when $H_{\rm inf}/2\pi$ falls below f_a/N. In this case, one expects axionic strings to be diluted during the remaining inflationary stage, and the relic density will be suppressed by an additional factor, $\exp(N_{\rm SSB})$, where $N_{\rm SSB}$ is the number of e-folds between PQ symmetry breaking and the end of inflation. As a consequence, this density should be negligible today, unless $N_{\rm SSB}$ is fine-tuned to small values by assuming that f_a/N is very close to the Hubble rate at the end of inflation.

- *before inflation:* if, for instance, the PQ symmetry is restored at very high energy and breaks before inflation is turned on e.g. at low scales, the axionic strings produced in that way, as well as the possible axions into which they may have decayed, will be diluted by inflation and can be safely neglected.

Note, however, that, apart from axionic strings, there are also axionic domain walls bounded by axionic strings. In the case that the PQ color anomaly corresponds to $N = 1$, the network of domain walls and strings decay away. However, for $N > 1$, the domain walls end up dominating the energy density of the universe, contrary to observations [30]. Solving this problem requires breaking the N degeneracy, by slightly lowering the energy of one of the N vacua and thus inducing the decay of the domain walls. However, this possibility seems far fetched from the point of view of model building, and we will ignore it here. A detailed discussion is given in Ref. [31]. From now on, we will assume that $N = 1$. We will also assume that the reheating temperature after inflation is not high enough to restore the PQ symmetry and thus reproduce the mesh of axionic strings. Thus we are left with only one production mechanism, misalignment.

2.3 Generation via misalignment angle

This production mechanism takes place at very early stages in the universe. When the PQ symmetry is explicitly broken, the phase of the field Θ may or may not be at the minimum of its potential. As explained above, a (or Θ) is a massless field during inflation and thus it fluctuates quantum-mechanically. If the typical amplitude of quantum fluctuations is large enough, Θ could take different values in different points of our observable universe after inflation, with a flat probability distribution in the range $[-\pi, \pi]$; otherwise, it could remain nearly homogeneous. In both cases, at the time of the QCD transition, the (local or global) value of the misalignment angle Θ_1 can differ from zero, leading to the sudden appearance of a potential energy term,

$$\begin{aligned}\rho_{\rm a} &= \frac{1}{2}\dot{a}^2 + \frac{(\vec{\nabla}a)^2}{2R^2} + m_a^2 f_a^2 (1-\cos\Theta)\ , \\ &\simeq \frac{1}{2} f_a^2 \left(\dot{\Theta}^2 + \frac{(\vec{\nabla}\Theta)^2}{R^2} + m_a^2\Theta^2\right) \quad \text{for small } \Theta. \end{aligned} \tag{2.13}$$

Actually, the gradient energy can be safely neglected in Eq. (2.13). Indeed, even in the case in which the axion is maximally inhomogeneous, i.e. when the phase Θ is equally distributed in the range $[-\pi, \pi]$ in our observable universe at the end of inflation, it is straightforward to show that at any later time the coherence length (the physical size of the "homogeneity patches" for a) is always of the order of the Hubble radius. This can be checked e.g. by solving the equation of motion (2.5) in Fourier space. As a consequence, and recalling that a is defined in the range $[-\pi f_a, \pi f_a]$, the typical size of the gradient $\vec{\nabla}a$ is given by $f_a H$. So, the gradient energy scales as $H^2 \propto R^{-4}$ during radiation domination, and at the time of the QCD transition it is at most of the order of $(f_a H_{\rm QCD})^2$. A quick estimate gives $H_{\rm QCD} \sim 10^{-11}$eV, while in the following we will always consider values of the axion mass much larger than this. So, when the axion mass is "switched on", the gradient energy is negligible with respect to the potential energy $V \sim (f_a m_a)^2$.

Since $m_a(T)$ grows suddenly to values much bigger than $H_{\rm QCD}$, one can also deduce from Eq. (2.5) that after acquiring its mass the field quickly rolls down towards the minimum of the potential, since the condition for rolling is $m_a(T) \gg 3H$. After a lapse of time that depends on the initial value of the misalignment angle, Θ_1, the field will reach the lowest energy point and start oscillating. For the latest stages of the oscillation we have $m_a(T) \gg 3H$ (and also $m_a(T) \simeq m_a$) so that the expression

$$\rho_a = \left\langle \frac{1}{2} f_a^2 \left(\dot{\Theta}^2 + m_a^2 \Theta^2 \right) \right\rangle = \frac{1}{2} f_a^2 m_a^2 \left\langle \Theta_1^2 \right\rangle \left(\frac{R_{\rm QCD}}{R} \right)^3 \tag{2.14}$$

becomes a good approximation for the spatial average of the axion density over our observable Universe.

However, for a precise estimate, it is necessary to take into account the time dependence of the mass when solving Eq. (2.5). In Ref. [32] the evolution equation is numerically solved for the case $m_a(T) \propto T^{-p}$, and it is found that (2.14) is corrected by a factor $f_c(p) \simeq 0.44 + 0.25p$, that we will take into account in the final computation of the energy density. After the QCD transition, the axionic energy density evolves with time as

$$\rho_a \frac{R^3}{m_a(T)} = \text{const.}\,, \tag{2.15}$$

where R is the scale factor of the universe. While the energy density varies with the temperature, the number density $n_a = \rho_a / m_a(T)$ is conserved in a comoving volume, which simply reflects axion number conservation. Thus, the current energy density of axions is related to the number density $n_{a,1}$ at the time t_1 where oscillations start by

$$\rho_a^{\rm today} = \gamma \frac{s_0}{s_1} m_a \, n_{a,1}\,, \tag{2.16}$$

where $s = (2\pi^2/45) g_* T^3$ is the entropy density, g_* counts the relativistic degrees of freedom present in the universe at a given temperature, and γ accounts for a possible entropy release after the axion starts oscillating [33]. Solving for T_1 from the condition $m_a(T_1) \simeq 3H(T_1)$, we find a relic axion density

$$\Omega_{\rm a} h^2 \simeq 7.24 \, g_{*,1}^{-5/12} \langle \Theta_1^2 \rangle \left(\frac{200 \text{MeV}}{\Lambda_{\rm QCD}} \right)^{\frac{3}{4}} \left(\frac{1 \mu\text{eV}}{m_a} \right)^{\frac{7}{6}} . \tag{2.17}$$

If the axion begins oscillating roughly at the time when it acquires a mass, i.e. when the temperature of the universe is slightly larger than $\Lambda_{\rm QCD}$, then $g_{*,1} = 61.75$. In Eq. (2.17), the spatial average $\langle \Theta_1^2 \rangle$ of the initial misalignment squared angle is not given by any field theoretical reasoning; rather its value should be inferred from some inflationary initial conditions and statistical remarks, see below.

2.4 Quantum diffusion of the axion during inflation

Let us assume that the Peccei-Quinn symmetry was spontaneously broken above the energy scale of inflation, corresponding to the length scales that we observe in the CMB today.

Thus we need to take into account the effect of the de Sitter quantum fluctuations [34] that induce a stochastic diffusion of the axion away from the initial value Θ_i.

The probability $\mathcal{P}(\Theta, N_e)$ of finding a certain value Θ at a time given by the number of e-folds N_e satisfies a Fokker-Planck equation with diffusion coefficient $D = H_{\rm inf}/(2\pi f_a)$,

$$\frac{\partial \mathcal{P}}{\partial N_e} = \frac{1}{2} D^2 \frac{\partial^2 \mathcal{P}}{\partial \Theta^2}\,, \tag{2.18}$$

whose solution is

$$\mathcal{P}(\Theta, N_e) = \frac{1}{\sqrt{2\pi D^2 N_e}} \exp\left[-\frac{(\Theta - \Theta_i)^2}{2D^2 N_e}\right]\,. \tag{2.19}$$

Thus, as pointed out in [17, 18, 20], a given inflationary domain that starts at a particular initial value Θ_i will be dispersed, after N_e e-folds of inflation, by

$$\langle (\Theta - \Theta_i)^2 \rangle^{1/2} = \frac{H_{\rm inf}}{2\pi f_a} \times \sqrt{N_e}\,. \tag{2.20}$$

That is, the field will have randomly walked a distance proportional to the square root of the number of jumps, in this case given by the number of e-folds.

The Fokker-Planck equation provides a good description of the stochastic evolution of the phase at a given point in real space; however, it does not give any hint on the spatial stucture and on the coherence length of the axion field. A straighforward analysis based on Fourier space reveals that the coherence length (or "scale of homogeneity") of any light or massless field during inflation at a given time is given by the Hubble radius at that time. Since the comoving Hubble radius $1/(aH)$ decreases with time during inflation, the stochastic evolution caused by quantum fluctuations can be seen as a process of fragmentation of an initially nearly homogeneous domain into several smaller nearly homogeneous patches [20].

In the present context, this remark is crucial, because it allows to identify the number of e-folds N_e which is really relevant in equation (2.20). When the scale corresponding to the observable universe crosses the Hubble length during inflation (i.e., typically, between 30 and 70 e-folds before the end of inflation), the axion field is nearly homogeneous over a length comparable to the size of the whole observable universe. Therefore, we should count the number of jumps *starting from that moment* (there might be many e-folds of inflation before that time, but they are only relevant for comparing the axion field value in our observable universe with that in other unaccessible, disconnected universes). Let us call the corresponding number of e-folds $N_{\rm obs}$, and from now on, let us define Θ_i as the average value of Θ inside the observable universe $N_{\rm obs}$ e-folds before inflation ending.

At the end of inflation, the coherence length of the axion field is smaller than the size of the observable universe by approximately a factor $\exp[N_{\rm obs}]$. In other words, the initial domain of average value Θ_i has fragmented into many domains of average value $\Theta(\vec{x})$, with a dispersion $\langle (\Theta - \Theta_i)^2 \rangle^{1/2}$ given by equation (2.20) with $N_e = N_{\rm obs}$.

Later on, causal diffusion tends to smooth the axion field over the Hubble length. Since after inflation, the comoving Hubble radius $1/(aH)$ increases with time, the number of homogeneity patches inside the observable universe decreases (after reaching its maximum at the end of inflation). In the next section, we will evaluate the dispersion of the phase Θ at the time of the QCD transition. By that time, fluctuations on scales smaller than the Hubble radius $H_{\rm QCD}^{-1}$ have been washed out. Therefore, the relevant dispersion $\langle(\Theta - \Theta_i)^2\rangle^{1/2}$ is given by the number of jumps during inflation between $N_{\rm obs}$ and $N_{\rm QCD}$, the time of Hubble exit for the comoving scale $k_{\rm QCD}$ which re-enters the horizon when $H = H_{\rm QCD}$. The number of e-folds between $N_{\rm obs}$ and $N_{\rm QCD}$ is given roughly by

$$\Delta N = \ln \frac{a_{\rm QCD} H_{\rm QCD}}{a_0 H_0} \sim \ln \frac{\Lambda_{\rm QCD}}{\Lambda_0} \sim 30. \tag{2.21}$$

So, the dispersion of the PQ phase at the time of the QCD transition and inside the observable universe is given by [17, 18, 20]

$$\langle(\Theta - \Theta_i)^2\rangle^{1/2} \sim \frac{H_{\rm inf}}{2\pi f_a} \times \sqrt{30}\,. \tag{2.22}$$

At this point, we see that two situations can occur. First, if $f_a \gg H_{\rm inf}$, the right-hand-side in equation (2.20) can be much smaller than one at the end of inflation; then, the axion field is essentially homogeneous, and the background value Θ_1 in our universe is random but unique. Second, if $f_a \leq H_{\rm inf}$, the right-hand-side can be of order one or larger, which means that the Brownian diffusion of the axion is complete, and the misalignment angle at the QCD scale is randomly distributed with a flat probability distribution in the range $[-\pi, \pi]$. Note that in this case, the quantum perturbations of the radial part of the PQ field are also large. In both cases, the mean energy density of the axion is proportional to

$$\langle\Theta_1^2\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi} \alpha^2 d\alpha = \frac{\pi^2}{3}\,, \tag{2.23}$$

where the average should be understood as holding over many realizations of the universe in the case of a nearly homogeneous Θ_1, or over our present Hubble radius in the case of complete diffusion.

Up until this point we have ignored the anharmonic corrections that could arise from the possibly large value of Θ_1. The calculations have been made using the approximation $(1 - \cos\Theta) \simeq \frac{1}{2}\Theta^2$, which is obviously not valid for large angles. To take into account this uncertainty, we will use $\langle\Theta_1^2 f(\Theta_1)\rangle$ instead of the plain rms value of Θ_1. When included, one finds $\langle\Theta_1^2 f(\Theta_1)\rangle \simeq 1.2\,\pi^2/3$.

3. Isocurvature perturbations from axion fluctuations

If the PQ symmetry is spontaneously broken during inflation, while the scale of inflation is much higher than that of the quark-hadron transition, the flat direction associated with the massless Nambu-Goldstone boson will be sensitive to de Sitter quantum fluctuations. Indeed, quantum fluctuations are imprinted into every massless scalar field present during inflation, with a nearly scale invariant spectrum, see [35],

$$\langle|\delta a(k)|^2\rangle = \left(\frac{H_{\rm inf}}{2\pi}\right)^2 \frac{1}{k^3/2\pi^2}\,. \tag{3.1}$$

If the scale of inflation is high enough, it is possible that quantum fluctuations of the radial part of the PQ field will restore the symmetry, for $H_{\rm inf}/2\pi > f_a$ [20]. However, in that case, the effective potential for the radial part (in fact for both the real and imaginary parts) has a mass-squared at the false vacuum $r = 0$ which is given by $m^2_{\rm eff} = H^2_{\rm inf}/48\pi^2 \ll H^2_{\rm inf}$, and thus behaves like a light field during inflation. Its quantum fluctuations become frozen beyond the horizon and thus leave a long wave perturbation, still described by equation (3.1).

The axion field perturbations δa do not perturb the total energy density, first because the potential energy is exactly zero, and second because as explained before, the gradient energy of the axion cannot exceed $\sim (f_a H_{\rm inf})^2$; for $f_a \ll M_P$ this is much smaller than the total energy density $(3/8\pi)M_P^2 H^2_{\rm inf}$. Since the total energy density, and thus the curvature, are unperturbed by these perturbations during inflation, they are of *isocurvature* type, and manifest themselves as fluctuations in the number density of axions [12, 13, 14, 15, 16],

$$\delta \left(\frac{n_a}{s} \right) \neq 0\,. \tag{3.2}$$

In the absence of thermal symmetry restoration after inflation, i.e. if the temperature of the plasma does not reach f_a, the axion does not couple significantly to ordinary matter. Thus the fluctuations imprinted during inflation still do not change the total curvature/density of the universe on super-Hubble scales during radiation domination, but they may contribute to the temperature anisotropies of the CMB [36, 37, 38].

Once the axions acquire a mass, it is in principle possible that their interactions with other particles could drive them into thermal equilibrium, thus converting their isocurvature perturbations into curvature or density perturbations. However, the axion coupling is so tiny that it effectively remains decoupled from the rest of the plasma throughout the history of the universe, even after acquiring a mass at the QCD transition. It cannot thermalize and this means that whatever fluctuations the axion has imprinted from inflation are truly isocurvature [39].

We will study here a scenario in which the observable power spectra of CMB anisotropies and large scale structures are given by the sum of two contributions: an adiabatic mode seeded by the inflaton perturbation, and an isocurvature mode seeded by the axion perturbations. Since these two fields have independent quantum fluctuations during inflation, the adiabatic and isocurvature modes are totally uncorrelated.

Let us assume that the Universe contains photons (γ), approximately massless neutrinos (ν), baryons (b), axions (a), ordinary CDM such as neutralinos (x) and a cosmological constant. In the following, the subscript cdm will denote the total cold dark matter component, so that $\Omega_{\rm cdm} = \Omega_{\rm a} + \Omega_{\rm x}$.

- For the mode seeded by the inflaton, the perturbation evolution starts from the initial condition (during radiation domination and on super-Hubble modes) $\frac{3}{4}\delta_\gamma = \frac{3}{4}\delta_\nu = \delta_{\rm b} = \delta_{\rm x}$, while $\delta_{\rm a} = 0$. The perturbations in the relativistic components $\delta_\gamma = \delta_\nu$ are themselves related to the curvature perturbation $\mathcal{R}$. Since below the QCD scale the two types of cold dark matter share the same equation of state, they are equivalent to a single fluid obeying to the initial condition $\delta_{\rm cdm} = R_{\rm a}\delta_{\rm a} + (1 - R_{\rm a})\delta_{\rm x}$, with

$R_{\rm a} \equiv \Omega_{\rm a}/\Omega_{\rm cdm}$. For the mode under consideration, $\delta_{\rm a} = 0$ and $\delta_{\rm x} = \delta_{\rm b}$, so that $\delta_{\rm cdm} = (1 - R_{\rm a})\delta_{\rm b}$. This initial condition is different from the standard adiabatic initial condition in a model without axions, $\delta_{\rm cdm} = \delta_{\rm b}$. However, this initial condition for $\delta_{\rm cdm}$ is irrelevant in practise, because cold dark matter perturbations do not leave a direct signature in the CMB anisotropies, while the observable matter power spectrum today is sensitive to the initial value of $\mathcal{R}$ but not of $\delta_{\rm cdm}$. This can be explicitely checked by running a Boltzman code with the standard adiabatic initial condition replaced by $\delta_{\rm cdm} = (1 - R_{\rm a})\delta_{\rm b}$: one finds that for fixed curvature spectrum and fixed $\Omega_{\rm cdm}$, the CMB and LSS power spectra do not depend on $R_{\rm a}$. Therefore, the mode seeded by the inflaton is equivalent to the usual adiabatic mode.

- For the isocurvature mode, the perturbation evolution starts from the initial condition $\frac{3}{4}\delta_\gamma = \frac{3}{4}\delta_\nu = \delta_{\rm b} = \delta_{\rm x} \simeq 0$ and $\delta_{\rm a} = \mathcal{S}_{\rm a}$, where $\mathcal{S}_{\rm a}$ is the gauge invariant entropy perturbation
$$\mathcal{S}_{\rm a} = \frac{\delta(n_{\rm a}/s)}{(n_{\rm a}/s)} = \frac{\delta n_{\rm a}}{n_{\rm a}} - 3\frac{\delta T}{T} \tag{3.3}$$
(indeed, after the QCD transition, the axion fluid is non-relativistic with $\rho_{\rm a} = m_{\rm a} n_{\rm a}$, so $\delta_{\rm a} = (\delta n_{\rm a})/n_{\rm a}$; furthermore, the fact that $\delta_{\rm a} \gg \delta_\gamma$ implies $(\delta n_{\rm a})/n_{\rm a} \gg 4(\delta T)/T$ and $S_{\rm a} = (\delta n_{\rm a})/n_{\rm a} = \delta_a$). Again, it is equivalent to consider the perturbations of a single cold dark matter fluid, obeying now to the initial condition $\delta_{\rm cdm} = R_{\rm a}\delta_{\rm a} + (1 - R_{\rm a})\delta_x = R_{\rm a}\mathcal{S}_{\rm a}$. If we compare with the initial condition for a usual "Cold Dark matter Isocurvature" (CDI) model, given by $\delta_{\rm cdm} = \mathcal{S}_{\rm cdm}$, we see that the axionic isocurvature solution is equivalent to the CDI solution with $\mathcal{S}_{\rm cdm} = R_{\rm a}\mathcal{S}_{\rm a}$.

In summary, an axionic model with axionic fraction $R_{\rm a} = \Omega_{\rm a}/\Omega_{\rm cdm}$, initial curvature spectrum $\langle \mathcal{R}^2 \rangle$ and initial entropy spectrum $\langle \mathcal{S}_{\rm a}^2 \rangle$ is strictly equivalent to a mixed adiabatic+CDI model with the same curvature spectrum and $\langle \mathcal{S}_{\rm cdm}^2 \rangle = R_{\rm a}^2 \langle \mathcal{S}_{\rm a}^2 \rangle$.

Let us now relate the curvature and entropy power spectrum to the quantum fluctuations of the inflaton and axion field during inflation. For the adiabatic mode, it is well-known that the curvature power spectrum reads

$$\langle |\mathcal{R}(k)|^2 \rangle = \frac{2\pi H_k^2}{k^3 M_P^2 \epsilon_k} \tag{3.4}$$

where ϵ is the first inflationary slow-roll parameter [35] and the subscript k indicates that quantities are evaluated during inflation, when $k = aH$. In first approximation this spectrum is a power-law with a tilt $n_{\rm ad}$ depending also on the second slow-roll parameter η [35],
$$n_{\rm ad} = 1 - 6\epsilon_k + 2\eta_k \ . \tag{3.5}$$

For the isocurvature mode, using the axion perturbation spectrum of Eq. (3.1), we obtain

$$\langle |\mathcal{S}_{\rm a}(k)|^2 \rangle = \left\langle \left| \frac{\delta n_{\rm a}}{n_{\rm a}} \right|^2 \right\rangle = 4 \left\langle \left| \frac{\delta a}{a} \right|^2 \right\rangle = \frac{2H_k^2}{k^3 f_a^2 \langle \Theta_1^2 \rangle} \, . \tag{3.6}$$

This power spectrum can be approximated by a power-law with a tilt

$$n_{\rm iso} = 1 - 2\epsilon_k \, , \tag{3.7}$$

which is related to the tilt n_t of tensor perturbations.

The relative amplitude of isocurvature perturbations at a given pivot scale in adiabatic+CDI models is often parametrized as [40, 41, 42, 43]

$$\alpha = \frac{\langle |\mathcal{S}_{\rm cdm}(k)|^2 \rangle}{\langle |\mathcal{S}_{\rm cdm}(k)|^2 \rangle + \langle |\mathcal{R}(k)|^2 \rangle} \; . \tag{3.8}$$

Since the axionic model is equivalent to an adiabatic+CDI model, we can still use the same parametrization. The parameter α is related to fundamental parameters by

$$\alpha = \frac{R_{\rm a}^2 \langle |\mathcal{S}_{\rm a}(k)|^2 \rangle}{R_{\rm a}^2 \langle |\mathcal{S}_{\rm a}(k)|^2 \rangle + \langle |\mathcal{R}(k)|^2 \rangle} \simeq \frac{R_{\rm a}^2 M_P^2 \epsilon_k}{\pi f_{\rm a}^2 \langle \Theta_1^2 \rangle} \tag{3.9}$$

where in the last equality we assumed $\alpha \ll 1$.

4. The axionic window

We now describe the generic constraints on f_a coming from different cosmological and astrophysical considerations. Some of them are generic and apply to every production mechanism or inflationary scenario while some others are rather model dependent. More precisely, we will enumerate the different bounds on the axion parameter space ($M_{\rm inf}$, f_a) associated with the misalignment mechanism of axion production during inflation, $M_{\rm inf}$ being the energy scale during inflation

$$M_{\rm inf} = \sqrt{\frac{M_{\rm P} H_{\rm inf}}{\sqrt{8\pi/3}}} \, , \tag{4.1}$$

and $M_{\rm P} \equiv G^{-1/2}$ is the Planck mass. The explicit implications of these bounds are presented on Figs. 2, 3.

4.1 The scale of inflation

The non-detection of tensor modes [25] imposes a constraint on the inflationary scale. The current bound on the tensor to scalar ratio is:

$$r \sim \left(\frac{4 \times 10^4 \, H_{\rm inf}}{M_{\rm P}} \right)^2 < 0.3 \quad \text{at } 95\% \text{ c.l.} \, , \tag{4.2}$$

which sets an upper bound on the inflationary scale:

$$M_{\rm inf} < 3 \times 10^{16} \text{ GeV} \, . \tag{4.3}$$

In Figs. 2, 3, this constraint corresponds to the hatched forbidden region.

4.2 Supernova 1987A bounds

The observed neutrino luminosity from supernova 1987A imposes a bound on the axion luminosity that is saturated for $10^{-2}\,\mathrm{eV} < m_a < 2\,\mathrm{eV}$ [5]. Other astrophysical and laboratory searches rule out an axion heavier than 1 eV (see [1, 5] for a detailed discussion). Therefore, we have:

$$m_a < 10^{-2} - 10^{-3}\ \mathrm{eV}\,, \tag{4.4}$$

or equivalently, $f_a > 10^9 - 10^{10}$ GeV. The forbidden region is (blue-)shaded in Figs. 2, 3.

4.3 Axionic cosmic strings production

As mentioned before, in the case of symmetry restoration at high energy, axions can be produced from the decay of cosmic strings appearing at the end of inflation [20] or after reheating [19]. Then, we must impose that the corresponding relic density does not exceed the total cold dark matter density,

$$\Omega_{\mathrm{a}}^{\mathrm{str}} \leq \Omega_{\mathrm{cdm}}\ , \tag{4.5}$$

where $\Omega_{\mathrm{a}}^{\mathrm{str}}$ is taken from Eq. (2.11). This inequality must be imposed in two cases: $f_a > H_{\mathrm{inf}}/2\pi$, corresponding to symmetry restoration during inflation[1], and $f_a > T_{\mathrm{rh}}$, corresponding to symmetry restoration after reheating. Actually, it is worth mentioning that thermal corrections induce a positive mass-squared term $m^2_{\mathrm{eff}} = T^2_{\mathrm{rh}}/12$, so the precise condition for symmetry restoration is

$$T^2_{\mathrm{rh}}/12 \gg \lambda f_a^2\ . \tag{4.6}$$

Therefore, the usual statement that symmetry is restored whenever $f_a > T_{\mathrm{rh}}$ assumes that the self-coupling constant λ is of the order of 0.1. We will proceed with this assumption, but one should keep in mind that the exact condition is model-dependent.

The actual reheating temperature T_{rh} is still unknown. This is the temperature at which the inflaton decays, once its half life has been exceeded by the age of the universe, that is, when $H \sim \Gamma$. Most of the thermal energy comes from perturbative decays of the inflaton and, assuming that the decay products are strongly interacting at high energies, we can estimate the reheating temperature as

$$T_{\mathrm{rh}} \simeq 0.1\,\sqrt{\Gamma M_P} \simeq 0.02\,h_{\mathrm{eff}}\,\sqrt{m\,M_P} \leq 2\times 10^{11}\ \mathrm{GeV}\,, \tag{4.7}$$

where $\Gamma = h^2_{\mathrm{eff}} m/8\pi$ is the inflaton decay rate, which is typically proportional to the inflaton mass, m, with $h_{\mathrm{eff}} \leq 10^{-3}$, in order to prevent radiative corrections from spoiling the required flatness of the inflaton potential [34]. This estimate shows the generic inefficiency of reheating after inflation, where the scale of inflation could be of order 10^{15} GeV and the reheating temperature ends being many orders of magnitude lower. For instance, for

[1] Note that for simplicity, we impose this condition as if H_{inf} was constant at least during the observable e-folds of inflation (typically, the last sixty e-folds). In principle, the amplitude of quantum fluctuations $H_{\mathrm{inf}}/2\pi$ could fall below f_a precisely during the observable e-folds, see e.g. [13], but we will ignore this possibility here.

Starobinsky type inflation, the weak gravitational couplings give a reheating temperature of order $T_{\rm rh} \sim 10^9$ GeV, while in chaotic inflation models, typical values are of order $10^{10} - 10^{11}$ GeV. We can parametrize the effect on the rate of expansion by introducing an efficiency parameter, $\epsilon_{\rm eff}$, such that $H_{\rm rh} = \epsilon_{\rm eff}\, H_{\rm end}$. Values of ϵ range from 10^{-13} for Starobinsky inflation, to order one for very low scale inflation.

Within certain low scale inflationary models, such as hybrid inflation, the efficiency of reheating can be significant because the rate of expansion at the end of inflation is much smaller than any other mass scale and the inflaton decays before the universe has time to expand, therefore all the inflaton energy density gets converted into radiation. If the reheating temperature is higher than f_a it could eventually lead to a restoration of the PQ symmetry. The subsequent spontaneous symmetry breaking would generate axionic cosmic strings that would not be diluted away by inflation.

In summary, if the PQ symmetry is restored by thermal fluctuations after reheating, i.e.

$$T_{\rm rh} = 0.1\sqrt{H_{\rm rh} M_{\rm P}} = 0.1\sqrt{\epsilon_{\rm eff} H_{\rm inf} M_{\rm P}} > f_a\,, \tag{4.8}$$

then we must impose the condition (4.5). In Figs. 2, 3, we have distinguished two cases: one in which the process of reheating the universe is very inefficient ($\epsilon_{\rm eff} \leq 10^{-12}$), and there is no thermal restoration of the PQ symmetry after inflation, and another one in which $\epsilon_{\rm eff} = 10^{-4}$ so that the symmetry might be restored. In both cases the constraint coming from string production and decay corresponds to the triangular (red-)shaded exclusion region.

4.4 Cold Dark Matter produced by misalignment

The extremely low values for the coupling of axions to other particle species make it a plausible candidate for cold dark matter. Therefore, constrains on this parameter must be applied to the relic energy density of axions. From Eq. (2.17) we have

$$\omega_{\rm a} \equiv \Omega_a\, h^2 \simeq 1.3\langle\Theta_1^2 f(\Theta_1)\rangle \left(\frac{1\,\mu{\rm eV}}{m_a}\right)^{7/6} = 2.8\times 10^7\langle\Theta_1^2 f(\Theta_1)\rangle \left(\frac{f_a}{M_{\rm P}}\right)^{7/6} \lesssim\ \omega_{\rm cdm}\,. \tag{4.9}$$

The bounds on $\omega_{\rm cdm}$ from our analysis and considerations about the particular inflationary model put severe constrains on the axion window. First, the inequality (4.9) provides a stringent upper limit on f_a if $\langle\Theta_1^2 f(\Theta_1)\rangle$ is of order one. In particular, in the case of complete quantum diffusion during inflation, we have seen that $\langle\Theta_1^2 f(\Theta_1)\rangle = 1.2\,\pi^2/3$ and

$$f_a \leq 2.5\times 10^{11}{\rm GeV}\left(\frac{\omega_{\rm cdm}}{0.12}\right)^{6/7}. \tag{4.10}$$

This constraint is shown in Figs. 2, 3 as a dotted line, excluding the light green region.

In the absence of efficient quantum diffusion, Θ_1 could take any nearly homogeneous value in our Universe: so, it is possible in principle to assume that $\langle\Theta_1^2 f(\Theta_1)\rangle$ is extremely small (this coincidence can be justified by anthropic considerations[2]), and to relax the

[2]There has been plausible speculations that our presence in the universe may not be uncorrelated with the values of the fundamental parameters in our theories. Such anthropic arguments normally arise in terms

bound on f_a. However, the mean square cannot be fine-tuned to be smaller than the amplitude of quantum fluctuations at the end of inflation. Using Eq. (2.20), we see that $\langle\Theta_1^2\rangle$ can only vary within the range

$$\left(\frac{H_{\rm inf}}{2\pi f_a}\right)^2 \Delta N < \langle\Theta_1^2\rangle < \frac{\pi^2}{3} \ , \tag{4.12}$$

where $\Delta N \sim 30$ is the number of inflationary e-folds associated with scales that reentered the Hubble radius when the QCD transition took place, see Eq. (2.21). This gives a model-independent constraint

$$M_{\rm inf} \leq 2.5 \times 10^{15}{\rm GeV} \left(\frac{\omega_{\rm cdm}}{0.12}\right)^{1/4} \left(\frac{30}{\Delta N}\right)^{1/4} \left(\frac{f_a}{10^{12}{\rm GeV}}\right)^{5/24} \tag{4.13}$$

which holds only in the region where

$$\left(\frac{H_{\rm inf}}{2\pi f_a}\right)^2 \Delta N < \frac{\pi^2}{3} \ , \tag{4.14}$$

otherwise it should be replaced by (4.10). This bound excludes the dark green region in Figs. 2, 3.

4.5 Isocurvature modes

As mentioned before, the axionic field induces an isocurvature component in the CMB anisotropies that must be considered when constraining the model. In this work, we assume that axions are the only source of isocurvature modes. Taking expression (3.9) for the isocurvature fraction α, replacing $R_{\rm a}$ by $\omega_{\rm a}/\omega_{\rm cdm}$ and using Eq. (4.9), we obtain

$$\alpha = \frac{0.9 \times 10^7 \epsilon_k}{\omega_{\rm cdm}^2} \left(\frac{M_{\rm P}}{f_a}\right)^{5/6} \ . \tag{4.15}$$

The analysis of the next section will provide bounds on α and $\omega_{\rm cdm}$. Also, it will give constraints on the curvature power spectrum $\langle|\mathcal{R}(k)|^2\rangle$, from which one can derive a relation between ϵ_k and $M_{\rm inf}$, using equation (3.4). Therefore, the amplitude of isocurvature modes provides some independent constraints in the $(M_{\rm inf},\, f_a)$ plane, corresponding to the central (yellow-)shaded forbidden region in Figs. 2, 3.

We should distinguish here between two cases:

---

of conditional probability distributions of particular observables. In particular, the axion abundance is a natural parameter that could be bounded by those arguments, see e.g. Refs. [13, 44] where it is suggested that the initial misalignment angle should be such that the main CDM component be axionic. In this case, one has a concrete prediction for $R_{\rm a} = \Omega_{\rm a}/\Omega_{\rm cdm} = 1$, and therefore the initial misalignment angle is directly related to the axion mass, see Eq. (4.9),

$$m_a = 9\,\mu{\rm eV}\,\langle\Theta_1^2 f(\Theta_1)\rangle^{6/7} \, . \tag{4.11}$$

Having full diffusion, $\langle\Theta_1^2 f(\Theta_1)\rangle \sim 1.2\pi^2/3$, implies $m_a \sim 30\,\mu$eV, just within reach of present axion dark matter experiments. On the other hand, we might happen to live in an unusual region of the universe with an extremely low value of $\langle\Theta_1^2\rangle$, a large value of f_a and still $R_a = 1$.

- **Quantum de Sitter fluctuations induce PQ symmetry restoration during inflation**. We have already seen that in this case, the real and imaginary parts of the PQ behave like light fields during inflation, thus leaving a long wave isocurvature perturbation on the axion field. In this case, the above constraint from CMB anisotropies is applicable.

- **Thermal fluctuations induce PQ symmetry restoration after reheating**. If the reheating temperature is much higher than f_a (more precisely, than $\sqrt{12\lambda}f_a$, see Eq.(4.6)), one expects thermal fluctuations to modify the effective PQ potential and to induce a large positive mass-squared term. In this case, the radial part is quickly driven to zero everywhere in the universe. All previous axion fluctuations are erased and we are left with no bounds from axion isocurvature temperature anisotropies. So, for sufficiently large $\epsilon_{\rm eff}$, the isocurvature constraint does not apply above a given line in the $(M_{\rm inf},\ f_a)$ plane, and the allowed region is split in two parts (as in Fig. 3).

5. Results

The aim of this work is to tighten the current allowed range for f_a by computing the allowed cold dark matter and isocurvature contribution given several sets of data described below.

To explore the axionic isocurvature contribution to the CMB anisotropies power spectrum, we assumed a flat ΛCDM universe, with 3 species of massless neutrinos and seven free parameters, with the same notation as in Ref. [41]: ω_b, the physical baryon density, $\omega_{\rm cdm}$, the total cold dark matter density (an arbitrary fraction of which is made of axions), θ, the angular diameter of the sound horizon at decoupling, τ, the optical depth to reionization, $n_{\rm ad}$, the adiabatic spectral tilt, $A_s = \ln[10^{10}k^3\langle|\mathcal{R}|^2\rangle]$, the overall normalization of super-Hubble curvature perturbations during radiation domination at the pivot scale $k = 0.002$ Mpc^{-1} and α, the isocurvature contribution (we sampled from $|\alpha|$ instead to avoid boundary effects near the maximum likelihood region). Our data consists in CMB data (WMAP (TT, TE and EE) [25], VSA [45], CBI [46] and ACBAR [47]); large scale structure data (2dFGRS [48] and the SDSS [49]); and supernovae data [50, 51]. Note that previous studies [41] indicated a very weak sensitivity of this data to $n_{\rm iso}$, while in our model $n_{\rm iso} = 1-2\epsilon_k$ is very close to one. So, we safely fix $n_{\rm iso}$ to exactly one without modifying the results. A top-hat prior probability distribution was assigned to each parameter inside the ranges described in table 1.

We used the Metropolis-Hastings algorithm implemented by the publicly available code `CosmoMC` [52] to obtain 32 Monte Carlo Markov chains, getting a total of 1.1×10^5 samples. We find a $\chi^2/\mathrm{DOF} = 1.01$ and the worst variance of chain means over the mean of chain variances value is 1.04 [53].

The one dimensional posterior probability distributions for sampled and derived parameters are depicted in Fig. 1. In particular, the bestfit value for α is $\alpha = 8\times10^{-4}$, and the 2σ bound for the marginalized distribution is:

$$\alpha < 0.08 \text{ at } 95\% \text{ c.l} \tag{5.1}$$

Parameter	Prior probability range
$\omega_{\rm b}$	(0.016,0.030)
$\omega_{\rm cdm}$	(0.08,0.16)
θ	(1.0,1.1)
τ	(0.01,0.2)
$n_{\rm ad}$	(0.85,1.1)
A_s	(2.7,4.5)
$\lvert\alpha\rvert$	(-1,1)

Table 1: Prior probability ranges of the sampled parameters.

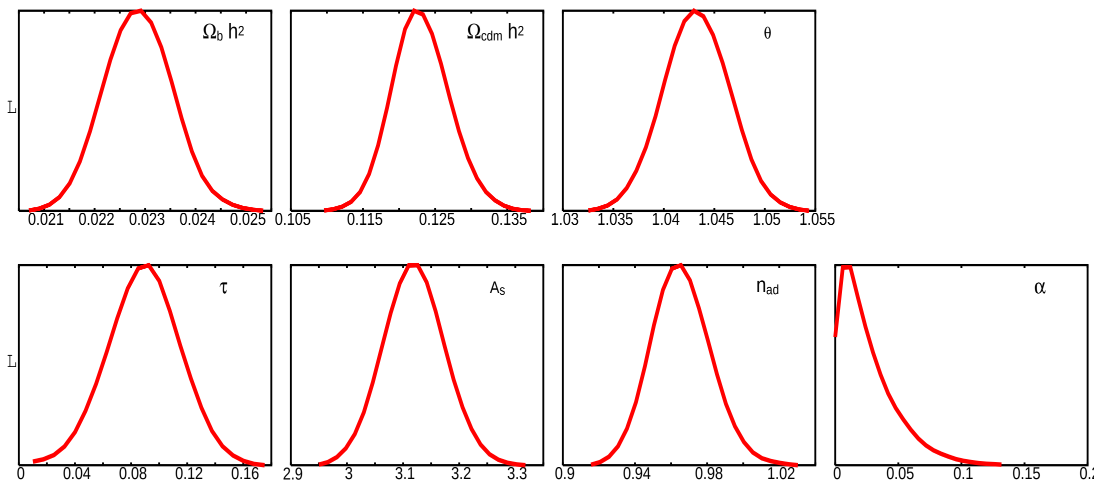

Figure 1: The one dimensional distributions of the sampled parameters.

while for $\Omega_{\rm cdm}\, h^2$ we get:

$$\Omega_{\rm cdm}\, h^2 = 0.123 \pm 0.008 \text{ at } 95\% \text{ c.l.} \tag{5.2}$$

5.1 A (nearly) model-independent analysis

At least two bounds in the $(f_a, M_{\rm inf})$ plane are completely model-independent: the upper bound $M_{\rm inf} < 3 \times 10^{16}$ GeV, coming from the non-observation of gravitational waves in the CMB (see Sec. 4.1) and the lower bound $f_a > 10^{10}$ GeV from astrophysical constraints (see Sec. 4.2).

When the PQ symmetry is restored at high energy, we must impose $\omega_{\rm a}^{\rm str} < 0.123$, where $\omega_{\rm a}^{\rm str}$ is taken from Eq. (2.11): this gives $f_a < 1.25 \times 10^{11}$ GeV. As explained in Secs. 4.3 and 4.5, this constraint applies when $H_{\rm inf}/2\pi > f_a$ during inflation and also when $T_{\rm rh} > f_a$ after reheating (the latter bound depends on $\epsilon_{\rm eff}$ in each particular model; it actually depends also on λ, see Eq. (4.6),but in this section as well as in the figures we assume for simplicity that $12\lambda \simeq 1$).

Even if there is no significant axion relic density from string decay, there is one associated with the misalignment angle mechanism. As explained in Sec. 4.4, the constraint $\omega_a < 0.123$ gives $f_a < 2.5 \times 10^{11}$ GeV, unless the misalignment angle is fine-tuned to very small values. In the latter case, there is still a much weaker model-independent bound

$$M_{\rm inf} \leq 2.5 \times 10^{15}{\rm GeV} \left(\frac{f_a}{10^{12}\,{\rm GeV}} \right)^{5/24} \quad . \tag{5.3}$$

All these bounds are summarized on Figs. 2, 3, with thick solid lines for model-independent constraints, and dotted lines for model-dependent ones.

Let us finally discuss the impact of the isocurvature mode limit $\alpha < 0.08$, which applies as long as the PQ symmetry is not restored by thermal corrections, $T_{\rm rh} < f_a$. Our results for the amplitude of the primordial curvature spectrum gives a relation

$$\epsilon_k \sim 3 \times 10^8 \left(\frac{M_{\rm inf}}{M_{\rm P}} \right)^4 \quad . \tag{5.4}$$

Substituting in Eq. (4.15), we see that the constraint $\alpha < 0.08$ finally gives

$$M_{\rm inf} \leq 10^{13}{\rm GeV} \left(\frac{f_a}{10^{12}\,{\rm GeV}} \right)^{5/24} \quad . \tag{5.5}$$

So, the isocurvature mode limit excludes a large region in parameter space, and *preserves only two regions*. The first one is present only if reheating is efficient enough, $\epsilon_{\rm eff} \geq 4 \times 10^{-12}$, and obeys to[3]

$$10^{10}{\rm GeV} < f_a < 1.2 \times 10^{11}\,{\rm GeV} \,, \qquad \frac{6 \times 10^{10}\,{\rm GeV}}{\sqrt{\epsilon_{\rm eff}}} < M_{\rm inf} < 3 \times 10^{16}{\rm GeV} \,. \tag{5.6}$$

The second one, for which the PQ symmetry is broken during inflation but the isocurvature mode is too small to be excluded by current cosmological data, corresponds to

$$10^{10}{\rm GeV} < f_a < 2.5 \times 10^{11}\,{\rm GeV} \,, \qquad M_{\rm inf} < 8 \times 10^{12}\,{\rm GeV} \,, \tag{5.7}$$

where we assumed that the average misalignment angle in the observable universe is of order one: otherwise, the upper bound on f_a could be relaxed significantly, while that on $M_{\rm inf}$ would only increase slightly, as f_a to the power $5/24$, see Eq. (5.5).

For a large class of well-known inflationary models, $M_{\rm inf}$ is typically of the order of 10^{15} or 10^{16} GeV, and the relevant allowed window (if any) is the first one. In the next section, we feature the example of chaotic inflation with a quadratic potential. For these high-scale models, we see that the compatibility between the PQ axion scenario and inflation depends mainly on the reheating efficiency. However, the second window (5.7) is also relevant, since it is possible to build low-scale inflationary model compatible with WMAP constraints and satisfying $M_{\rm inf} < 8 \times 10^{12}$ GeV. Actually, the main motivation for the original hybrid inflation model of Ref. [15] was precisely the possibility to evade

[3]Here, we still are still assuming for simplicity that λ is of the order of 0.1. If this is not the case, the correct result is obtained by replacing the factor $\epsilon_{\rm eff}$ by $\epsilon_{\rm eff}/(12\lambda)$.

axionic isocurvature constraints. Nowadays, many low scale inflation models can be built in the generic framework of hybrid inflation (see Ref. [55] for a review, or [56] for a recent example). In this case, the isocurvature perturbations carried by the axion are to small to be constrained by current data, and under the conditions of (5.7) the PQ axion model is perfectly viable.

5.2 Bounds for chaotic inflation with a quadratic potential

Inflation with a monomial potential $V(\phi) \propto \phi^\alpha$ is usually called chaotic inflation. The latest WMAP results combined with other data sets essentially rule out cases with $\alpha > 4$, while the quartic case $\alpha = 4$ is only in marginal agreement with the data. Therefore, in this section we only consider the case of a quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$, still favored by observational bounds.

Using the COBE normalization, it is possible to prove that the mass should be of order $m \sim 5 \times 10^{-8} M_{\mathrm{P}}$, and to show that N_e e-folds before the end of inflation,

$$\phi_e^2 = \frac{N_e}{2\pi} M_{\mathrm{P}}^2 \ . \tag{5.8}$$

In particular, between N_{obs} and N_{QCD}, the scale of inflation should be in the range

$$3 \times 10^{15}\mathrm{GeV} < M_{\mathrm{inf}} < 4 \times 10^{15}\mathrm{GeV} \ , \qquad 3 \times 10^{12}\,\mathrm{GeV} < H_{\mathrm{inf}} < 4 \times 10^{12}\,\mathrm{GeV} \ . \tag{5.9}$$

So, if f_a is close to 10^{10} or 10^{11} GeV, one has $H_{\mathrm{inf}}/2\pi \geq f_a$ and the PQ symmetry is broken by quantum fluctuations during inflation. Still, the constraint from isocurvature modes applies, unless the symmetry is restored later by thermal fluctuations. Reheating after chaotic inflation is expected to lead to a temperature T_{rh} of the order of 10^{10} or 10^{11} GeV, which is precisely the allowed range for f_a. So, if f_a is pushed down to 10^{10} GeV while T_{rh} is pushed up to 10^{11} GeV, a very brief stage of thermal restoration could occur, and could be sufficient for erasing isocurvature perturbations.[4] We conclude that the PQ axion model can be marginally reconciled with chaotic inflation if $f_a \sim T_{\mathrm{rh}} \sim 10^{10} - 10^{11}$ GeV.

We could hope to find a second allowed window in the case where f_a is very large and the misalignment angle is fine-tuned to a very small value (i.e., inside the light green region in Figs. 2, 3). Note that this is not possible, at least in the case of chaotic inflation. Indeed, if f_a is much bigger than 10^{11} GeV, thermal symmetry restoration after inflation cannot take place, and isocurvature bounds are applicable. Given the value of M_{inf} in Eq. (5.9), the isocurvature constraint (5.5) yields $f_a \geq 8 \times 10^{23}\mathrm{GeV} \gg M_{\mathrm{P}}$, which is not conceivable.

5.3 Possible loopholes

In this subsection we will explore those loopholes we have left open for the axion to be the dominant component of cold dark matter.

[4] The condition for symmetry restoration is weaker when $\lambda \ll 1$, see Eq.(4.6). So, if λ is not of the order of 0.1 but significantly smaller, the isocurvature constraint is more easily evaded.

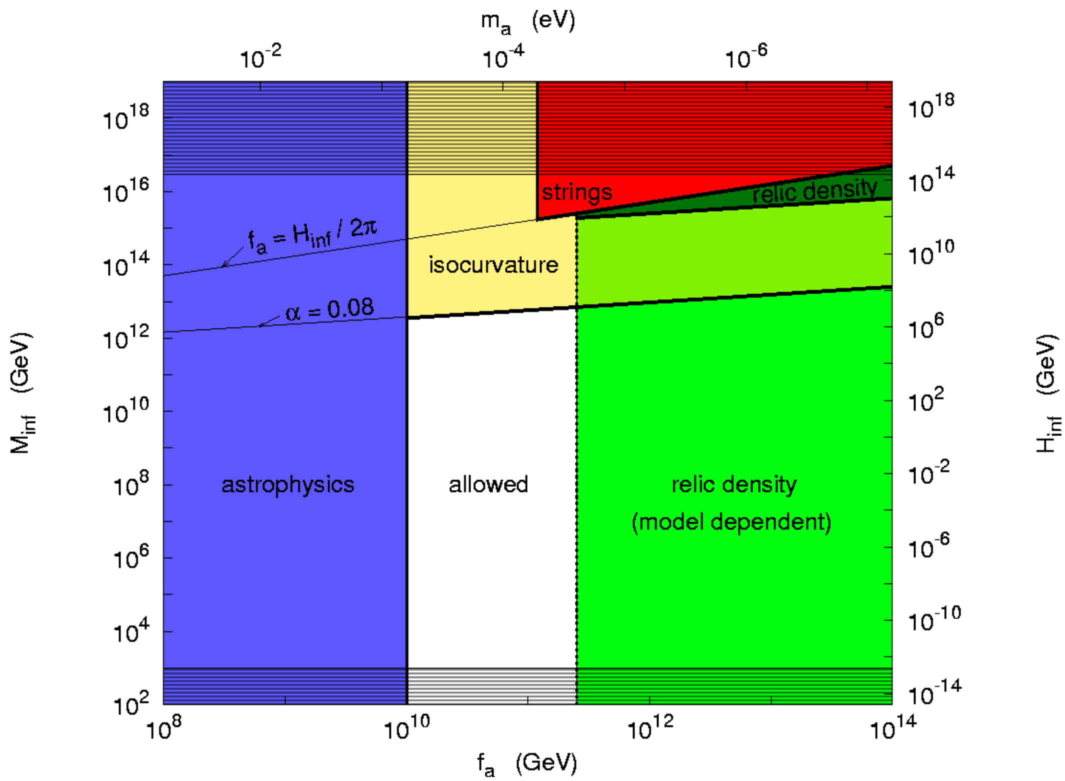

Figure 2: Bounds in the ($M_{\rm inf}$, f_a) plane, assuming that reheating is very inefficient and the PQ symmetry can never be thermally restored ($\epsilon_{\rm eff} \leq 4 \times 10^{-12}$). The energy scale of inflation $M_{\rm inf}$ is bounded from above by $r < 0.3$ (top hatched) and from below by the requirement of successful baryogenesis (bottom hatched). Possible values of the PQ scale f_a lie between the regions excluded by supernovae (left blue) and by the bound on the axionic relic density, produced by one of the following two mechanisms: axionic string decay (red) or misalignment angle at the time of the QCD transition (green). For the bounds related to the relic density, we use a solid line for the model independent bound (with a minimal, fine-tuned value of the misalignment angle), and a dotted line for the model dependent bound (with a generic misalignment angle of order one). The yellow region is excluded by the limit on the isocurvature mode amplitude α: this is the main result of this work. The remaining allowed region is left in white. Details are explained in the text.

5.3.1 Production of cosmic strings during reheating

Even if the reheating temperature of the universe is too low for a thermal phase transition at the Peccei-Quinn scale, it is possible that axionic cosmic strings be formed during preheating if the field responsible for symmetry breaking at the end of inflation is the Peccei-Quinn field. Then, the residual global $U(1)$ symmetry of the vaccum gives rise to axionic cosmic strings. At present there is no prediction for what is the scaling limit of such a mesh of strings produced during preheating. A crucial quantity that requires evaluation is the fraction of energy density in infinite strings produced at preheating, since they are the ones that will give the largest contribution to the axion energy density. It could then be that axionic strings produced at preheating may be responsible for the present axion

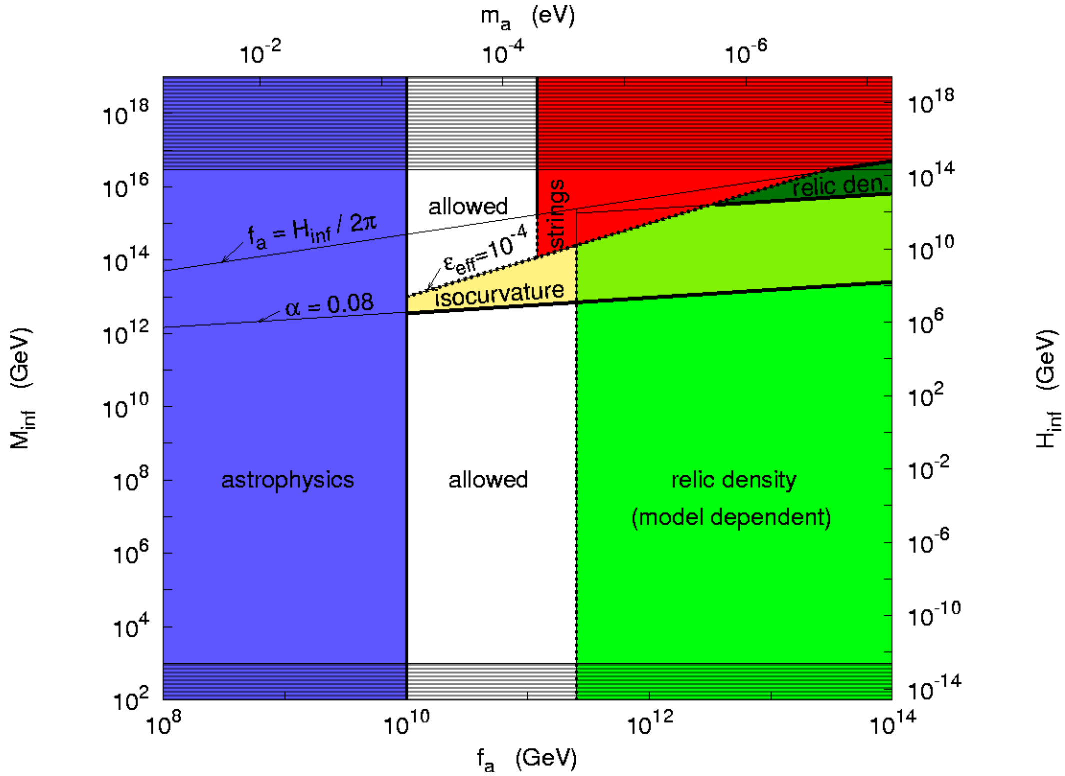

Figure 3: Same as previous figure, but assuming now that reheating is very efficient ($\epsilon_{\rm eff} = 10^{-4}$) so that the the PQ symmetry is thermally restored above the dotted line. In this case, the constraint from string production and decay (red) extends to smaller values of $M_{\rm inf}$, but above the dotted line and for $10^{10}\text{GeV} < f_a < 1.2 \times 10^{11}\text{GeV}$ there is a new allowed region (corresponding to the erasure of isocurvature perturbations in the thermal bath).

abundance. We leave for the future the investigation of this interesting possibility.

5.3.2 Axion dilution by late inflation

When imposing bounds on the axion mass from its present abundance, it is assumed that no there is no significant late entropy production or dilution from a secondary stage of inflation. We truly don't know. It has been speculated that a short period of inflation may be required for electroweak baryogenesis to proceed [57]. In such a case, a few e-folds ($N \sim 5$) of late inflation may dilute the actual axion energy density by a factor $\gamma = e^{-3N} \sim 10^{-7}$, easily evading the bounds.

6. Conclusions

In this paper, we reviewed various bounds on the cosmological scenario in which the cold dark matter is composed of axions plus some other component (like e.g. neutralinos), and the energy density of axions is produced by the misalignment mechanism at the time of the QCD transition. In our model, a fraction of the cosmological perturbations consists of isocurvature modes related to the quantum perturbations of the axion during inflation. We use this possible signature plus the contribution to dark matter energy density as a tracer of axions in the universe. In that sense, this work is similar to that presented in [58]. However, we make a stronger statement about the generation of isocurvature modes in the case in which the PQ symmetry is restored by quantum fluctuations during inflation, and we significantly improve previous bounds coming from the non-observation of isocurvature modes in the CMB, in particular, given the recent WMAP 3-year data.

A narrow window for the PQ scale $10^{10}\,\text{GeV} < f_a < 10^{12}$ GeV still remains open, but we show that the main consequence of recent data on cosmological perturbations is to limit the possible energy scale of inflation in this context, in order not to have an excessively large contribution of isocurvature perturbations to the CMB anisotropies. In particular, we show that in the axion scenario, the energy scale of inflation cannot exceed $M_{\text{inf}} < 10^{13}$ GeV, unless one of the two situations occurs: either reheating leads to a temperature $T_{\text{rh}} > f_a$, with an efficiency parameter $\epsilon_{\text{eff}} \geq 4\times 10^{-12}$, and there is another allowed region with $6\times 10^{10}\epsilon_{\text{eff}}^{-1/2}\,\text{GeV} < M_{\text{inf}} < 3\times 10^{16}$ GeV; or the misalignment angle is fine-tuned to very small values (this coincidence can be motivated by anthropic considerations) and the upper bound $M_{\text{inf}} < 10^{13}$ GeV can be slighly released (by at most one of magnitude).

As a consequence, if the axion scenario for CDM is valid, the chaotic inflationary model based on a quadratic potential is only marginally allowed. It requires f_a to be in the range $10^{10} - 10^{11}$ GeV, i.e. accessible to the next generation of axion search experiments, and the reheating temperature to be slightly above f_a. A combination of bounds from axionic string production and isocurvature perturbations completely rules out the case in which f_a would be much bigger than the above value, while the misalignment angle would have to be very small.

These bounds may be of interest given the present particle physics experiments searching for the axion, and this in turn may help to place stringent constraints on inflationary models. Detecting the QCD axion could shed some light on the scale of inflation and possibly into the mechanism responsible for inflation.

In this respect, there is an intriguing possibility that the PVLAS experiment [59] may have observed a pseudo-scalar particle coupled to photons. The nature of this particle is yet to be decided, since its properties seem in conflict with present bounds on the axion coupling to matter [9, 10], see however [60].

Acknowledgements.

It is a pleasure to thank Andrei Linde, David Lyth and Pierre Sikivie for enlightening discussions. We also thank A. Lewis for providing the updated CosmoMC code with the WMAP 3-year data. We acknowledge the use of the MareNostrum Supercomputer at the Barcelona Supercomputing Center. This work was supported in part by a CICYT project FPA2003-04597. The work of MB was also supported by the "Consejería de Educación de la Comunidad de Madrid–FPI program". We acknowledge the financial support of the CICYT-IN2P3 agreement FPA2005-9.

References

[1] See e.g. the reviews: J.E. Kim, Phys. Rep. **150** (1987) 1; H.-Y. Cheng, Phys. Rep. **158** (1988) 1; R.D. Peccei, in 'CP Violation', ed. by C. Jarlskog, World Scientific Publ., 1989, pp 503-551; M.S. Turner, Phys. Rep. **197** (1990) 67; G.G. Raffelt, Phys. Rep. **198** (1990) 1.

[2] R. D. Peccei and H. Quinn, Phys. Rev. Lett. **38** (1977) 1440; Phys. Rev. **D16** (1977) 1791.

[3] S. Weinberg, Phys. Rev. Lett. **40** (1978) 223; F. Wilczek, Phys. Rev. Lett. **40** (1978) 279.

[4] C. Vafa and E. Witten, Phys. Rev. Lett. **53** (1984) 535.

[5] Particle Data Group, http://pdg.lbl.gov

[6] S. Weinberg, Phys. Rev. Lett. **40** (1978) 223; W.A. Bardeen and S.-H.H. Tye, Phys. Lett. **B74** (1978) 229; J. Ellis and M.K. Gaillard, Nucl. Phys. **B150** (1979) 141; T.W. Donnelly et al., Phys. Rev. **D18** (1978) 1607; D.B. Kaplan, Nucl. Phys. **B260** (1985) 215; M. Srednicki, Nucl. Phys. **B260** (1985) 689.

[7] J. Kim, Phys. Rev. Lett. **43** (1979) 103; M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B166** (1980) 493.

[8] M. Dine, W. Fischler and M. Srednicki, Phys. Lett. **B104** (1981) 199; A. P. Zhitnitskii, Sov. J. Nucl. **31** (1980) 260.

[9] http://www.phys.ufl.edu/ axion/

[10] http://cast.web.cern.ch/CAST/

[11] K. Zioutas *et al.* [CAST Collaboration], Phys. Rev. Lett. **94** (2005) 121301; L. Duffy *et al.* [ADMX Collaboration], Phys. Rev. Lett. **95** (2005) 091304

[12] M. Axenides, R. H. Brandenberger and M. S. Turner, Phys. Lett. B **126** (1983) 178.

[13] A. D. Linde, JETP Lett. **40** (1984) 1333 [Pisma Zh. Eksp. Teor. Fiz. **40** (1984) 496]; Phys. Lett. B **158**, 375 (1985); Phys. Lett. B **201** (1988) 437.

[14] D. Seckel and M. S. Turner, Phys. Rev. D **32** (1985) 3178.

[15] A. D. Linde, Phys. Lett. B **259** (1991) 38.

[16] M. S. Turner and F. Wilczek, Phys. Rev. Lett. **66** (1991) 5.

[17] A. D. Linde and D. H. Lyth, Phys. Lett. B **246** (1990) 353.

[18] D. H. Lyth, Phys. Rev. D **45** (1992) 3394.

[19] D. Harari and P. Sikivie, Phys. Lett. **B195** (1987) 361; C. Hagmann and P. Sikivie, Nucl. Phys. **B363** (1991) 247; C. Hagmann, S. Chang and P. Sikivie, Phys. Rev. **D63** (2001) 125018.

[20] D. H. Lyth and E. D. Stewart, Phys. Rev. D **46** (1992) 532; Phys. Lett. **B283** (1992) 189.

[21] D. J. Gross, R. D. Pisarski and L. G. Yaffe, Rev. Mod. Phys. **53** (1981) 43.

[22] P. Fox, A. Pierce and S. D. Thomas, "Probing a QCD string axion with precision cosmological measurements," arXiv:hep-th/0409059.

[23] E. W. Kolb and M. S. Turner,"The Early Universe", Addison-Weseley (1990).

[24] E. Masso, F. Rota and G. Zsembinszki, Phys. Rev. D **66** (2002) 023004.

[25] D. N. Spergel *et al.*, "Wilkinson Microwave Anisotropy Probe (WMAP) three year results: Implications for cosmology," arXiv:astro-ph/0603449.

[26] S. Hannestad, A. Mirizzi and G. Raffelt, JCAP **0507** (2005) 002.

[27] R.A. Battye and E.P.S. Shellard, Phys. Rev. Lett. **73** (1994) 2954; E.P.S. Shellard and R.A. Battye, Phys. Rept. **307** (1998) 227; R.A. Battye and E.P.S. Shellard, Nucl. Phys. Proc. Suppl. **72** (1999) 88

[28] M. Yamaguchi, M. Kawasaki and J. Yokoyama, Phys. Rev. Lett. **82** (1999) 4578.

[29] A. S. Sakharov and M. Y. Khlopov, Phys. Atom. Nucl. **57**, 485 (1994) [Yad. Fiz. **57**, 514 (1994)]; A. S. Sakharov, D. D. Sokoloff and M. Y. Khlopov, Phys. Atom. Nucl. **59**, 1005 (1996) [Yad. Fiz. **59N6**, 1050 (1996)]; M. Y. Khlopov, A. S. Sakharov and D. D. Sokoloff, arXiv:hep-ph/9812286; Nucl. Phys. Proc. Suppl. **72** (1999) 105.

[30] P. Sikivie, Phys. Rev. Lett. **48** (1982) 1156.

[31] S. Chang, C. Hagmann and P. Sikivie, Phys. Rev. **D59** (1999) 023505.

[32] M. S. Turner, Phys. Rev. **D33** (1986) 889.

[33] P. J. Steinhardt and M. S. Turner, Phys. Lett. **B129** (1983) 51; G. Lazarides, R. Schaefer, D. Seckel and Q. Shafi, Nucl. Phys. **B346** (1990) 193.

[34] A.D. Linde, "Particle Physics and Inflationary Cosmology", Harwood Academic Press (1990).

[35] A. R. Liddle and D. H. Lyth, "Cosmological inflation and large-scale structure," Cambridge University Press (2000).

[36] D. Polarski and A. A. Starobinsky, Phys. Rev. D **50**, 6123 (1994); M. Sasaki and E. D. Stewart, Prog. Theor. Phys. **95**, 71 (1996); M. Sasaki and T. Tanaka, Prog. Theor. Phys. **99**, 763 (1998).

[37] J. García-Bellido and D. Wands, Phys. Rev. D **53**, 5437 (1996); **52**, 6739 (1995).

[38] C. Gordon, D. Wands, B. A. Bassett and R. Maartens, Phys. Rev. D **63**, 023506 (2001); N. Bartolo, S. Matarrese and A. Riotto, Phys. Rev. D **64**, 123504 (2001). D. Wands, N. Bartolo, S. Matarrese and A. Riotto, Phys. Rev. D **66**, 043520 (2002).

[39] S. Weinberg, Phys. Rev. D **70**, 083522 (2004).

[40] P. Crotty, J. García-Bellido, J. Lesgourgues and A. Riazuelo, Phys. Rev. Lett. **91** (2003) 171301.

[41] M. Beltrán, J. García-Bellido, J. Lesgourgues and A. Riazuelo, Phys. Rev. D **70**, 103530 (2004).

[42] M. Beltrán, J. García-Bellido, J. Lesgourgues, A. R. Liddle and A. Slosar, Phys. Rev. D **71** (2005) 063532.

[43] M. Beltrán, J. García-Bellido, J. Lesgourgues and M. Viel, Phys. Rev. D **72** (2005) 103515.

[44] M. Tegmark, A. Aguirre, M. Rees and F. Wilczek, Phys. Rev. D **73** (2006) 023505.

[45] R. Rebolo *et al.* [VSA Collaboration], Mon. Not. Roy. Astron. Soc. **353**, 747 (2004); C. Dickinson *et al.*, Mon. Not. Roy. Astron. Soc. **353**, 732 (2004).

[46] T. J. Pearson *et al.* [CBI Collaboration], Astrophys. J. **591**, 556 (2003); J. L. Sievers *et al.*, Astrophys. J. **591**, 599 (2003); A. C. S. Readhead *et al.*, Astrophys. J. **609**, 498 (2004).

[47] C. l. Kuo *et al.* [ACBAR Collaboration], Astrophys. J. **600**, 32 (2004); J. H. Goldstein *et al.*, Astrophys. J. **599**, 773 (2003).

[48] J. A. Peacock *et al.* [2dFGRS Collaboration], Nature **410**, 169 (2001); W. J. Percival *et al.*, Mon. Not. R. Astron. Soc. A **327**, 1297 (2001); **337**, 1068 (2002).

[49] M. Tegmark *et al.* [SDSS Collaboration], Astrophys. J. **606**, 702 (2004).

[50] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], "Measurements of Omega and Lambda from 42 High-Redshift Supernovae," Astrophys. J. **517** (1999) 565 [arXiv:astro-ph/9812133].

[51] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astrophys. J. **607**, 665 (2004).

[52] A. Lewis and S. Bridle, Phys. Rev. D **66**, 103511 (2002); CosmoMC home page: `http://www.cosmologist.info`

[53] S. Brooks and A. Gelman, JCGS, **7**:434-456 (1998).

[54] U. Seljak *et al.*, Phys. Rev. D **71**, 103515 (2005).

[55] D. H. Lyth and A. Riotto, Phys. Rept. **314** (1999) 1.

[56] R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, "Gauge invariant MSSM inflaton," arXiv:hep-ph/0605035.

[57] J. Garcia-Bellido, D. Y. Grigoriev, A. Kusenko and M. E. Shaposhnikov, Phys. Rev. D **60** (1999) 123504.

[58] P. Sikivie, "The search for dark matter axions," arXiv:hep-ph/0606014.

[59] E. Zavattini *et al.* [PVLAS Collaboration], arXiv:hep-ex/0512022.

[60] E. Masso and J. Redondo, JCAP **0509** (2005) 015.